# Cartan's Torsion: Necessity and Observational Evidence

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This article starts with the mathematical definition, concrete description, and physical meaning of Cartan's torsion. I proceed with the argumentation that torsion is required for the description of intrinsic spin. Moreover I argue that the duality between curvature and torsion is analogous to the duality between electricity and magnetism. I conclude this article by pointing out that the aligned rotation axes of the galaxies of the Perseus-Pisces supercluster may be interpreted as a topological defect generated by torsion.

## 1 What is Cartan's Torsion?

When a four-vector $C^k$ is parallely displaced from the four-position $x^k$ to $x^k + dx^k$, then it changes according to the prescription,

$$dC^k = -\Gamma^k_{ij}(x) C^j dx^i. \tag{1}$$

This is the definition for the position-dependent affine connection $\Gamma^k_{ij}$. According to general relativity [1], it has only a symmetric part,

$$\{\}^k_{ij} = \frac{1}{2}(\Gamma^k_{ij} + \Gamma^k_{ji}), \tag{2}$$

which is called "Christoffel symbol." The anti-symmetric part of the affine connection is called "Cartan's torsion" [2],

$$T^k_{ij} = \frac{1}{2}(\Gamma^k_{ij} - \Gamma^k_{ji}). \tag{3}$$

According to general relativity, the torsion tensor is zero. The introduction of a nonzero torsion tensor means therefore an extension of general relativity.

Quite remarkably, the torsion tensor transforms as a tensor under local Lorentz transformations, whereas the Christoffel symbol does not.

The torsion tensor can be viewed as the translational field strength. It represents a closure failure of infinitesimal displacements. In spacetimes which include torsion, infinitesimal parallelograms do not close.

We know from Einstein's general relativity [1] that gravitational mass is connected with curvature via

$$G^{ij} = \kappa \Sigma^{ij}, \tag{4}$$

where

$$G^{ij} = R^{ij} - \frac{1}{2} g^{ij} R^k_k \tag{5}$$

is the Einstein tensor, $\Sigma^{ij}$ is the stress-energy (energy-momentum) tensor, $R^{ij}$ is the Ricci tensor, $g^{ij}$ is the metric tensor, $R^k_k$ is the Ricci scalar, and $\kappa = -8\pi G/c^4$ is the Einstein constant.

Analogously, intrinsic spin is connected with Cartan's torsion via

$$T^{ijk} = \kappa \tau^{ijk}, \tag{6}$$

where $\tau^{ijk}$ is the spin tensor [3]. The equations (4) and (6) show the duality between mass and spin and between curvature and torsion, respectively.

Directly from the definition of the affine connection, Eq. (1), one obtains the differential equation of autoparallel curves,

$$\frac{d^2 x^k}{ds^2} + \Gamma^k_{ij} \frac{dx^i}{ds} \frac{dx^j}{ds} = 0, \tag{7}$$

where the infinitesimal interval $ds$ between $x^k$ and $x^k + dx^k$ is given by

$$ds^2 = g_{ij}(x) dx^i dx^j. \tag{8}$$

Quite remarkably, only the symmetric part of the metric tensor contributes to the square of the infinitesimal interval.

Readers who would like to learn more about the formalism of torsion are invited to read the excellent reviews, Ref. [4].

## 2 Why Do We Need Torsion?

The energy-momentum tensor $\Sigma^{ij}$ of a Dirac field $\Psi$ (spin 1/2 field [5]) is anti-symmetric [6],

$$\Sigma^{ij} = -\frac{\hbar c}{2} [(\nabla^i \bar{\Psi}) \gamma^j \Psi - \bar{\Psi} \gamma^j \nabla^i \Psi], \tag{9}$$

where

$$\nabla_i = \partial_i + ieA_i \tag{10}$$

is the covariant derivative. By contrast, the energy-momentum tensor of general relativity [1] is symmetric. In order to couple a spinor field (Dirac field) to a gravitational field, one has to use an energy-momentum tensor which includes

anti-symmetric parts. Therefore general relativity has to be generalized by the introduction of Cartan's torsion [3].

I have shown that the duality between mass and spin is analogous to the duality between electric charge and magnetic charge [7]. The electric-magnetic duality is,

$$J^i = \partial_j F^{ji} \tag{11}$$

$$j^i = \partial_j f^{ji}, \tag{12}$$

where $J^i$ is the electric four-current, $j^i$ is the magnetic four-current, and the field strength tensors are given by,

$$F^{ji} = \partial^j A^i - \partial^i A^j \tag{13}$$

$$f^{ji} = \partial^j a^i - \partial^i a^j, \tag{14}$$

where $A^j$ is the electric four-potential which corresponds to Einstein's electric photon [8], and $a^j$ is the magnetic four-potential which corresponds to Salam's magnetic photon [9].

Comparison of Eqs. (11) and (12) with Eqs. (4) and (6) demonstrates the analogy between the electric-magnetic duality and the mass-spin duality.

The electric-magnetic duality is required to explain the quantization of electric charge [10]. Quantum field theoretical models which include the magnetic photon can be found in Ref. [11]. I argued [12] that magnetic photon radiation may have already been observed by August Kundt in 1885 [13].

It is probably interesting to note that a Maxwell field $A^i$ which is coupled to a gravitational field which includes both Cartan's torsion [2] and Weyl's non-metricity [14], requires the appearence of the second four-potential $a^i$ [15].

Furthermore, Cartan's torsion tensor can be built from two independent vector fields which appear to obey the modified Maxwell equations of the two-photon theory [16].

# 3 Is There Observational Evidence for Torsion?

The rotation axes of the galaxies of the Perseus-Pisces supercluster are aligned. This alignment exists over a distance of at least 40 Mpc (130 million light years) [17]. Such a large alignment cannot be explained within the framework of conventional models of galaxy-formation. Therefore I suggested [18] that this alignment is either a topological defect (torsion wall [19]) or a remnant of the original aligned distribution of galactic rotation axes generated by a rotating universe [20]. My interpretation of this structure as a torsion wall [21] or as an effect of a rotating universe [22] is now generally accepted.

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